The lookback option pricing for Amazon based on Monte-Carlo Simulation

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Abstract: Among all the options of Amazon, Lookback options have lower risk compared with European options and relative higher return if under the same risk. Therefore, we investigated the Lookback option pricing for Amazon, to discuss the suitability of Lookback options for Amazon stocks. Based on Black-Scholes pricing model and Monte-Carlo simulations as well as data collected from Yahoo finance, the payoff of Lookback options and the sensitivity are demonstrated, respectively. According to the analysis, when the market volatility is high, the payoff of lookback options is higher than European options, while the profit may not higher than European options since the premium of lookback options are higher. There results shed light on the investment for Amazon Lookback options.

1. Introduction

Option is a contract that gives the holder the right to buy or sell an asset at a fixed price on a specific date or any time before that date. Generally, as stated by Mishkin [1], developed financial instruments such as futures, swaps and option can help bank to reduce risk rate but do not require bank to rearrange the balance sheet. The value of option is based on the assets purchased or sold, which includes stocks, government bonds, currencies, stock indexes, commodity future and other assets. Option belongs to a kind of derivative financial instrument as they are derivative of these assets. However, the option seller does not necessarily own the underlying assets in option. Option sellers can sell options short. Indeed, option holders may not want to buy the underlying assets in his option. Therefore, when the option expires, the two parties may not necessarily carry out the physical delivery of the subject matter, but only need to make up the price based on the spread to realize their contract.

Unlike other financial instrument, the risk condition of option is unpredictable and non-linear. Therefore, the factors (e.g., call &put off) sensitivity of option should be taken into consideration. In other word, we could choose an option which can bring us profit and lower the risk of loss as much as possible.

In our paper, we would like to introduce the lookback option, which is a stable choice for option holder because of its unique character. Typically, lookback option is a kind of exotic option that allows the holder to determine when to exercise their option [2]. The payoff of lookback option depends on the optimal (maximum or minimum) underlying assets' price occurring over the life of the option. This option allows holders to "look back" over time to determine the payoff, thus it is a relatively low risky option.

According to Yahoo Finance, Amazon is a high price stock, which means plenty of investors who are interested in Amazon can not afford to invest in it. As mentioned in Ref [3], "I'd like to see an investor who maybe can't buy a lot of Apple to be able to reflect their take the stock," said Kinahan, "and while the mini options are debuting on just five tickers, he thinks there is room for expansion to more high-priced equities in the future." From the Ref. [3], we could see the economist intend to provide investors more opportunity to enter market. Because the price of companies like amazon is

extremely high, investors need to satisfy the cash criteria, which means the investors also suffer a great risk of loss, and the loss will be more destructive for investors who is not rich but put a lot in the stock. With the help, the risk of loss is shrunk to some extent. On the hand, Mini option stimulate the economy of market because more investors enter the market.

Another option economist choose for amazon is Bullish Option Strategy. The option keeps credit of AMZN as profit. If AMZN has dropped below this strike, the investor may be assigned and required to buy the shares for an effective price. Downside risk is equal to that of a straight stock purchase, or unlimited to zero [4], i.e., the option has a risk. Nevertheless, one of the investors could reduce the risk of margin calls down the road by putting available cash.

Based on the previous literatures, lookback option is a better option compared with the mini option and Bullish option. though we cannot get a very high return. In this paper, we use Monte-Carlo method to simulate the future payoffs of lookback option for Amazon stock and compare it with the standard option (European option). Monte Carlo simulation is a collection of computational techniques for the (usually approximate) solution of mathematical problems, which make fundamental use of random samples [5]. Besides, the sensitivity between some important parameters and payoffs are carried out for lookback options. Then, the attributed reasons are found that why lookback options can be a better choice for investors to hedge against large fluctuations in the Amazon market price than European option.

The rest of paper is organized as follow. The Sec. 2 will introduce the data and method utilized in this paper that analyzes lookback option. The Sec. 3 includes the result obtained from analyzing data, the future discussion and constrains of our research. Finally, the Sec 4 gives a brief summary.

2. Data & Methods

We used monthly data for close price of Amazon stock from 9/1/2016 data to 9/1/2021, which is collected from Yahoo Finance web site. As shown in Fig. 1, One can see that stock close price generally show an upward trend.
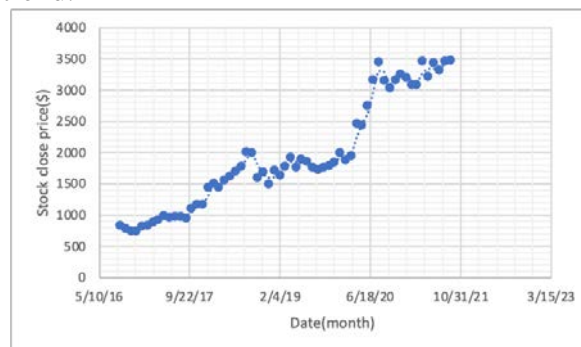

Fig. 1. The monthly close price for Amazon stock (9/1/2016-9/1/2021).

To price the Lookback option price, the following steps are used.

Step1: We use historical close price for Amazon stock to calculate the monthly returns and find the annually volatility.

Step2: Subsequently, we pick one of the close prices of stocks and fond risk free rate, dividend from yahoo finance. Then, we use some formula and those data to research the Lookback option pricing for Amazon. These formulae are included in the following steps.

Step3: Black-Scholes model is applied to find the future stock price for 7 days maturity. The data is grouped into one set every 7 days. We repeat this process 1000 times. The pricing formula for Black-Scholes model satisfies [6]:

$$S_T = S_0 e^{(\alpha - \frac{1}{2}\sigma^2)T + z\sigma\sqrt[2]{T}} \tag{1}$$

where S_T is the future value of an asset, S_0 is given the value of an asset now, T is the expiration time, z is standard normally distributed number, α is expected return, σ is standard deviation, σ^2 is

variance. Based on such a stochastic process, the expected return α and standard deviation are σ assumed constant at each moment in time, and the asset value change continuously. The formula can be used to compute a long range future price or a path of shorter term prices by changing the time interval.

Step4: Afterward, the Fixed Lookback Call Option Payoffs, Fixed Lookback Put Option Payoffs, Floating Lookback Call Option Payoffs, Floating Lookback Put Option Payoffs, European Call Option Payoffs, European Put Option Payoffs are calculated. Generally, there are 2 types of lookback options. We chose different formulas according to different calculation requirements. The first one is with Fixed strike price:

$$LCfix = Max(Smax - K , 0) \tag{2},$$

$$LPfix = Max(K – Smin, 0) \tag{3},$$

While the other one is with Floating strike price [7]:

$$LCfloat = Max(ST - Smin, 0) \tag{4},$$

$$LPfloat = Max(Smax - ST, 0) \tag{5}.$$

Here, the Smax represents for asset's maximum price during the life of the option, Smin is the asset's minimum price during the life of the option, ST denotes the asset's price at maturity T and K is the Strike price. As for European options, it obeys the formula [8]:

$$CT = Max(ST-K, 0) \tag{6}$$

where CT is the future value of an asset, ST is Asset's price at maturity T and K is Strike price. This means that until maturity date you can exercise the option.

Step5: Then, the Monte-Carlo simulations are carried out to calculate the future stock price for 1000 times [9, 10]. Monte-Carlo simulation means when the problem to be solved is the probability of the occurrence of a certain random event, or the expected value of a random variable, the probability of the occurrence of the event is estimated by the frequency of the occurrence of the event through some "experimental" method, or some digital features of the random variable that are obtained and taken as the solution of the problem.

Step6: Eventually, according to the calculation data and results, the graphs for the payoffs of lookback option, and graphs of the sensitivity to some important parameters are demonstrated.

3. Math Results & Discussion

3.1 Comparison between Lookback option & European option

After using Black-Scholes model and Monte-Carlo simulation, we find future payoff for1000 times of lookback option (with fixed strike price & with floating strike price) and European option. Then, the scatter diagram of the 1000 data for those two options is illustrated, presenting the relationship between future payoffs and stock price at maturity.

As shown in Figs. 2 and 3, the payoff of call for Fixed strike lookback option is more deviate with the change of stock price than its payoff of put. However, the payoff for call & put both have a clear trendline.
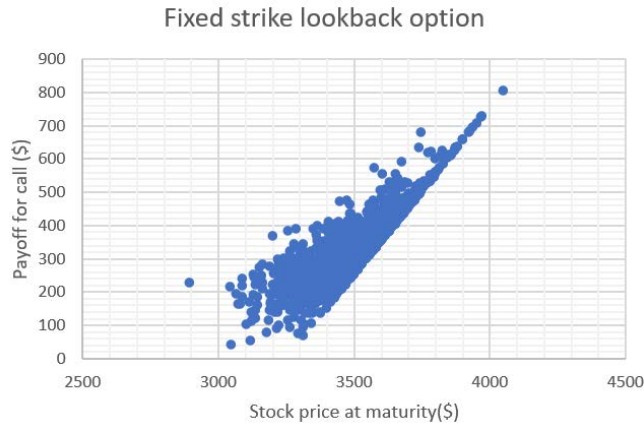
Fig. 2. The stock price for Amazon assets at maturity as a function of payoff of call for Fixed strike lookback option. Both with sign of dollar ($), maturity is 7 days.
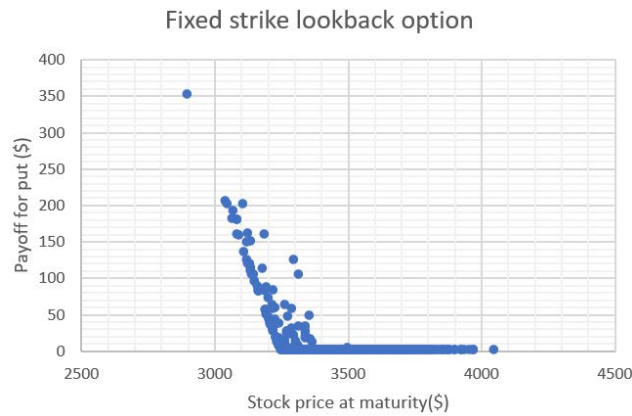

Fig. 3. Stock price for Amazon assets at maturitx as a function of the payoff of put for Fixed strike lookback option, y-axis be this y. Both with sign of dollar ($). Maturity is 7 days.

Whereas, as shown in Figs 4 and 5, the payoff of call & put for Floating strike lookback option does not have a clear trendline. Moreover, its payoff is much more deviate with the change of stock price than the payoff for Fixed strike lookback option.
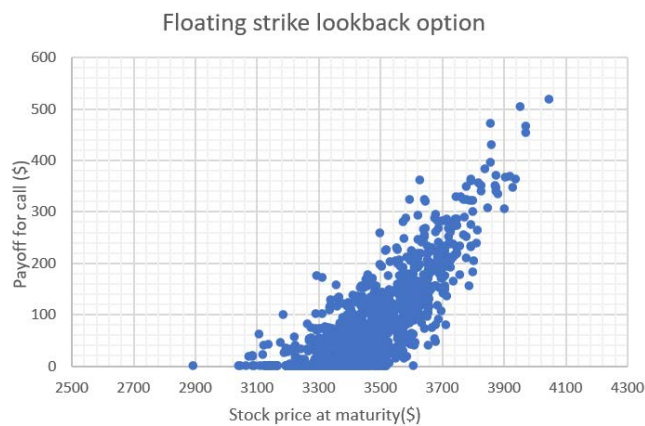

Fig. 4. The stock price for Amazon assets at maturity as a function of the payoff of call for Floating lookback option. Both with sign of dollar ($). Maturity is 7 days.
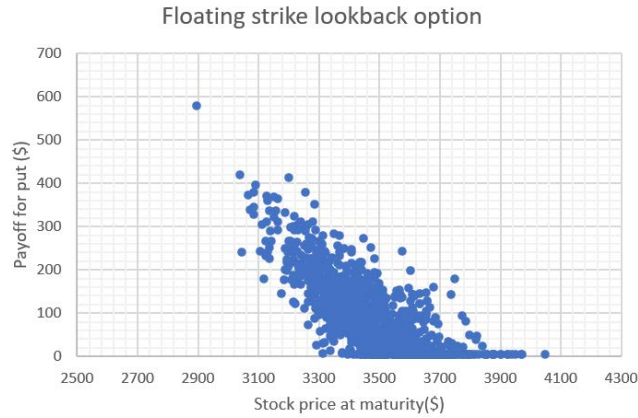
Fig. 5. The stock price for Amazon assets at maturity as a function of the payoff of put for Floating strike lookback option. Both with sign of dollar ($). Maturity is 7 days.

As shown in Figs. 6 and 7, the payoff for European option with the change of stock price has an extremely clear trendline and has no deviations on the trendline.

Fig. 6 The stock price for Amazon assets at maturity as a function of the payoff of call for European option. Both with sign of dollar ($). Maturity is 7 days.
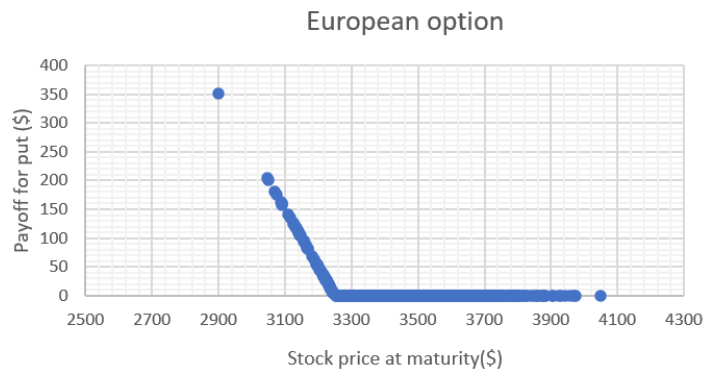

Fig. 7 The stock price for Amazon assets at maturity as a function of the payoff of call for European option. Both with sign of dollar ($). Maturity is 7 days.

So far, we have caught the clear sight of payoff diagram for Lookback option and European option. Then, we start to find their future price. The future price value of those options can be obtained from the arithmetic mean of the 1000 payoffs, by using the "Arithmetic mean" formula:

$$AM = \frac{1}{n}\sum_{i=1}^{n} a_i = \frac{a_1 + a_2 + \cdots + a_n}{n} \qquad (7)$$

where a_i denotes each number (where i=1, 2, … n), the arithmetic mean is the sum of all the numbers divided by n.

However, the future price value at maturity for those option is not the true value, due to time-value-of-money. Thus, we discount the average payoff from time T (time at maturity) to time 0 (Present) to get the present payoff for those options, using General Valuation Equation (GVE):

$$\text{Time Compensation+Risk Compensation} = \text{Expected capital gains} + \text{Cash flows} \qquad (8)$$

Here, we set time to be continuous and r to be continuously compounded. Then we derive a formula for pricing the present value of options from GVE [11]:

$$\lim_{n\to\infty} V\left(1 + \frac{r}{n}\right)^{-nT} = Ve^{-rT} \qquad (9)$$

where r is interest rate, T is time for maturity, and V represents for value at maturity

To only calculate the present value is not sufficient to do the comparison, we also do the calculation for the range of the present payoff. However, in order to gain the range, the value of standard deviation, standard error, and maximum error are necessary. Moreover, we have set the standard error be two standard deviations. As shown in Tables I and II, the average future price, the present value, the range for Lookback option and European option and some important factors have been illustrated.

Table I: The payoff at time 0 and time of maturity, the range of the payoffs, standard deviation, sample size, standard error of call for Lookback option and European option

Fixed strike lookback option		Floating strike lookback option		European option	
Call		Call		Call	
Estimate value of call at maturity		Estimate value of call at maturity		Estimate value of call at maturity	
Average at time T	312.45	Average at time T	88.37	Average at time T	237.71
Standard deviation	115.94	Standard deviation	89.60	Standard deviation	154.35
Sample	1000	Sample	1000	Sample	1000
Standard error	3.67	Standard error	2.83	Standard error	4.88
Maximum error(2sd)	7.33	Maximum error(2sd)	5.67	Maximum error(2sd)	9.76
At time 0		At time 0		At time 0	
Present value	321.45	Present value	88.36	Present value	237.71
Max error	7.33	Max error	5.67	Max error	9.76
Range	314.11	328.78	Range	82.70	94.03	Range	227.95	247.47

Table II. The payoff at time 0 and time of maturity, the range of the payoffs, standard deviation, sample size, standard error of put for Lookback option & European option

Fixed strike lookback option		Floating strike lookback option		European option	
Put		Put		Put	
Estimated value of put at maturity		Estimated value of put at maturity		Estimated value of put at maturity	
Average at time T	7.20	Average at time T	89.55	Average at time T	5.81
Standard deviation	28.50	Standard deviation	86.06	Standard deviation	26.44
Sample	1000	Sample	1000	Sample	1000
Standard error	0.90	Standard error	2.72	Standard error	0.84
Maximum error(2sd)	1.80	Maximum error(2sd)	5.44	Maximum error(2sd)	1.67
At time 0		At time 0		At time 0	
Present value	7.20	Present value	89.55	Present value	5.81
Max error	1.80	Max error	5.44	Max error	1.67
Range	5.40	9.00	Range	84.11	94.99	Range	4.14	7.48

Table III. strike price is set be 3250. The more precise value for present payoffs, max error, and range of payoff of call& put for Lookback option & European option

	Payoff for Call					
	Lookback option				European option	
	Fixed strike		Floating strike			
More accurate average		318.90		88.47		234.65
Max error		0.73		0.57		0.98
Range	318.17	319.64	87.91	89.04	233.67	235.63
	Payoff for Put					
	Lookback option				European option	
	Fixed strike		Floating strike			
More accurate average		6.38		89.10		4.85
Max error		0.18		0.54		0.17
Range	6.20	6.56	88.55	89.64	4.68	5.01

To be more precise about pricing, we apply Data Table in Excel for 100 times, meaning the simulations is acted 100 times more than past 1000 data (100*1000 frequency).

As shown in Table. III, the clear comparison of option value between Lookback option and European option is gained. Now, we only consider the value comparison between Fixed strike lookback option and European option. The call value is $318.9 and range from $318.17 to $319.64, with max error, for Fixed strike lookback option. For European option, the call value is $234.65 and range from $233.67 to $235.63, which has a lower call option value than Fixed strike lookback option. Moreover, the put option value for Fixed strike lookback option is also higher than that for European option. Since, the put value is $6.38 and range from $6.2 to $6.56, with max error, for Fixed strike lookback option. For European option, the put value is $4.85 and range from $4.68 to $5.01. Thus, we can conclude that Fixed strike lookback option has higher option value than European option.

Subsequently, we scrutinize Floating strike lookback option. Its call value is $88.47 and range from $87.91 to $89.04. Additionally, its put value is $89.1 and range from $88.55 to $89.64. When those values compared with European option's values, one knows the call value is lower and put value is higher for Floating lookback option. Nonetheless, the call value and put value for Floating lookback option is almost the same. This circumstance can be explained by the formula for pricing the Floating strike lookback option. According to Eqs. (2) and (3), the pricing for Floating strike lookback option only involved the maximum and minimum asset price during the lifetime of the option, and the price at maturity, but not the strike price. More specifically, it is affected by the fluctuation of the asset's prices during the life span. Thus, with the same fluctuation of the assets market, the payoff of call & put for Floating strike lookback option will have similar results, no matter what the strike price is. We will attest this analysis in the following paper using more apparent figures.

3.2 Influential factors for option value

To capture the changes of option value for Lookback option and European option with respect to spot price, we set different spot prices between $2800 and $3700 with increment $50 and stabilize the other factor. Then, we use Data Table to simulate and draw the results with scatter diagram in Excel. Call value is shown in Fig. 8, and put value is shown in Fig. 9. The relationship between option value and the spot price is represented by the trendline linear equation. The slope for the trendlines shows us the accurate changes of payoff with respect to spot price, which is the coefficient of the binary first order equation.
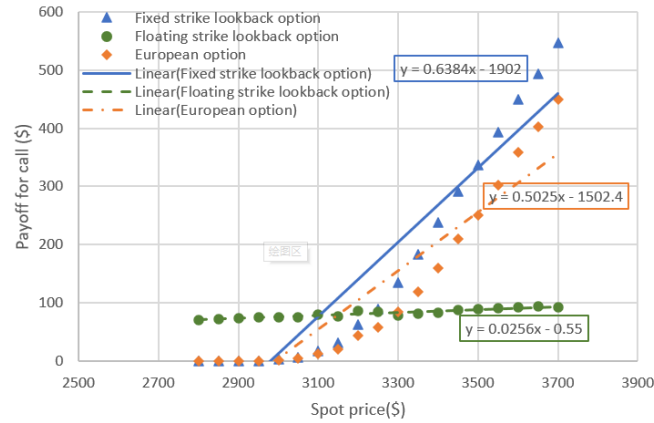
Fig. 8. Call value's sensitivity to spot price for lookback option & European option.

As shown in Fig, 8, when the spot price goes up by $1, the call value for Fixed strike lookback goes up by $0.6384 and the call value for European option goes up by $0.5025. Thus, when the spot price increases, call value for Fixed lookback option is increasingly higher than that for European option. As for floating lookback option, the slope is slowly increase and is approximately 0, indicating that there is almost no relationship between its payoff and the spot price. Therefore, even when the spot price is relatively high or is relatively low, the payoff for Floating strike lookback option can always be stable.
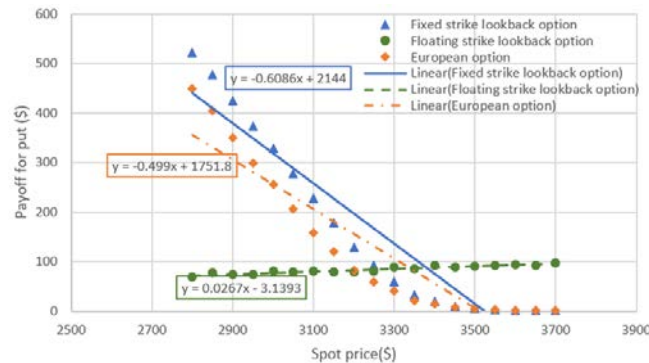

Fig. 9. Put value's sensitivity to spot price for Lookback option & European option.

As shown in Fig. 9, when the spot price goes down by $1, the put value for Fixed strike lookback goes up by $0.6086 and the put value for European option goes up by $0.499. Thus, when the spot price decreases, the put value for Fixed lookback option is increasingly higher that of European option. For Floating lookback option, the situation is similar with its call value. The slope is slowly increase and can approximately equal to 0. Hence, even if there is large change in spot price, the put value for Floating strike lookback option can always maintain similar values.

In order to gain the relationship between option value and strike price for lookback option and European option, the similar method is applied for spot price, except the variables and the invariants. We change strike price from $2800 to $3700 with increments $50 and leave the other variables unchanged. Call value is shown in Fig. 10, and put value is shown in Fig. 11.

For call in Fig. 10, when strike price goes down by $1, the call value for Fixed lookback option goes up by $0.9033, the payoff for European option goes up by $0.7971. Afterward, one concludes that when strike price decreases, the call value for Fixed lookback option is increasing higher than European option. In addition, when the strike price is high, there is no significant difference between the payoff of those two options.

However, for Floating lookback option, the slope is approximately 0. Therefore, this slope and the figure further proves our assumptions in the previous section that there is no relationship between the changes of call value and strike price for floating strike lookback.

For put in Fig. 11, when price goes up by $1, the put value for Fixed lookback option goes up by $0.2973, the put value for European option goes up by $0.2119. Therefore, it is convincible to state

that when strike price is relatively high, the put value for Fixed lookback option is higher than that for European option. Furthermore, if the strike price is low enough, put value for Fixed strike lookback option and European option will have similar results.

From the slope and figure, THE payoffs for Floating lookback option are not affected by the strike price. With the other variables unchanged, the put value for Floating lookback option is also unchanged.
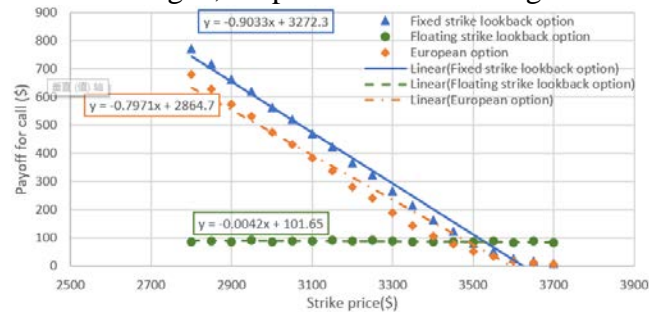

Fig. 10. Call value's sensitivity to strike price for Lookback option & European option
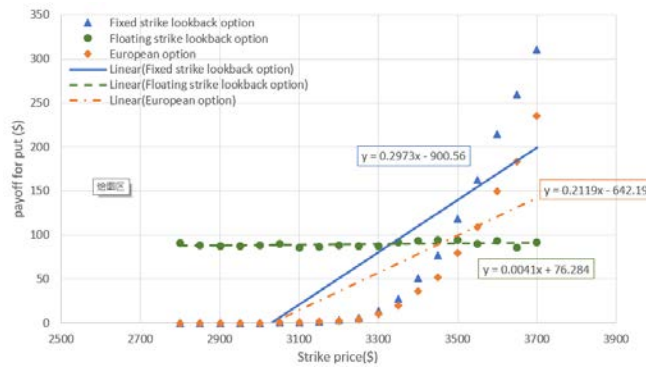

Fig. 11. Put value's sensitivity to strike price for Lookback option & European option

Regarding to volatility, we changed the volatility from 0.15 to 0.4 with increments 0.01 with other variables unchanged, apply Data table to simulate, draw scatter diagram for the simulations, and then get the option value's sensitivity to volatility for Lookback option and European option. Call value is shown in Fig. 12, and put value is shown in Fig. 13.

As shown in Fig. 12, the payoffs of Lookback option and European option are all increases with the increasing of volatility. The slope from the trendline linear equation shows us a more accurate increasements for the payoff. When volatility goes up by 0.01 percent, the payoff for Fixed strike lookback option goes up by $318.98, the payoff for Floating strike lookback option goes up by $317.42, the payoff for European option goes up by $65.187. Therefore, it is stated that lookback option is more sensitive to volatility. The higher the fluctuation is, the higher payoff for lookback option is.

As illustrated in Fig. 13, the put value of Lookback option and European option are also increases with the increasing of volatility. When volatility goes up by 0.01 percent, the payoff for Fixed strike lookback option goes up by $97.04, the put value for Floating strike lookback option goes up by $313.54, the call value for European option goes up by $69.918. Those number shows us that the call value put value for Floating strike lookback option is much more sensitive to volatility than European option. Moreover, even though the put value for Fixed strike lookback option is more sensitive to volatility than European option, their sensitivity does not different as much.

In brief, one can conclude that if the Amazon market has a large fluctuation, choosing lookback option can get a higher payoff than the standard European option.
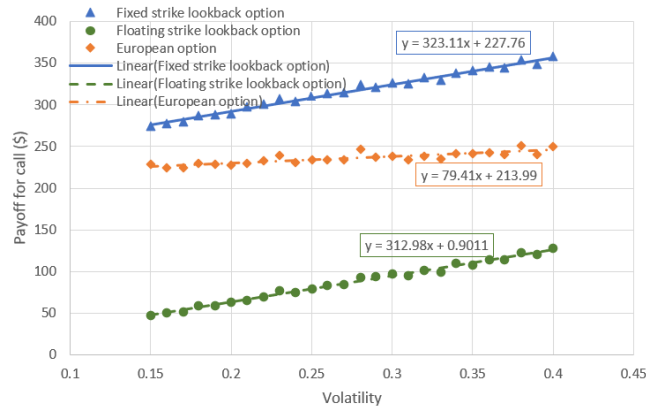
Fig. 12 Call value's sensitivity to volatility Lookback option & European option.
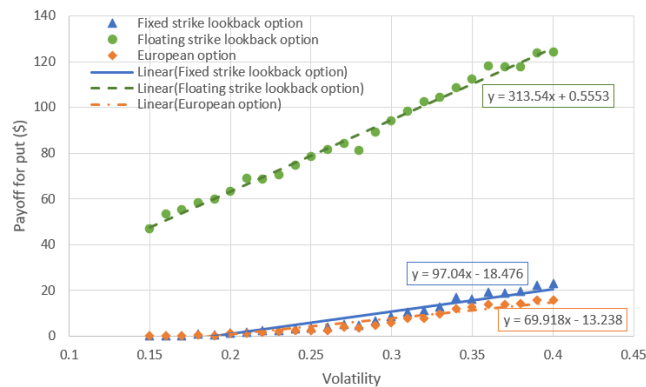

Fig. 13 Put value's sensitivity to volatility for Lookback option & European option

3.3 Limitations

There are three constrains in our analyze. First of all, the data we used is historical. Although the stimulation can reflect some common points of lookback option, the stock price changes every day owing to various impacts. Therefore, we cannot argue that the future's return can be absolutely the same as the data and graphs we get in above result. Secondly, the historical data used in our stimulation are limited. However, the stimulation is unlimited, thus there will exist some deviation. Finally, because the data we used is collected from Yahoo finance daily database, i.e., it is different from using continuously. Besides, here the premium of Amazon is set to zero since there isn't any premium of related before.

The statement we provided is for recommendation, Investment is risky, investors must be cautious when entering the market

4. Conclusion

In summary, we investigate the Lookback option pricing for Amazon with the data collected from yahoo finance website. According to Lookback option pricing model, Black-Scholes model, fixed strike price formula, floating strike price formula, Monte-Carlo Simulation, we obtained the corresponding results. Based on the analysis, we presented graphs for the payoffs of lookback option and graphs of the sensitivity to some important parameters to demonstrate the results. Specifically, one can see the payoff of Fixed lookback options are higher than the payoff of European options. Nevertheless, because strike price of Floating lookback options is always floating, the payoff of Floating lookback options can not be compared with the payoff of European options. When stocks' volatility is high, investors using Fixed lookback options can get higher payoffs than using European options. Therefore, lookback options will be a good choice to reduce risks and get higher payoffs for investors, especially for investors who have a lot of assets to invest. However, the study still has some limitations, because the insufficient amount of data, the future cannot be accurately predicted, and

there are many uncontrollable factors that affect the result. In the future, multiple computations ought to be carried out in order to make the result more precise. These results provide investors with comprehensive consideration, which helps reduce investment risk.

5. Conflict of Interest

The authors declare no conflict of interest.

6. Author Contributions

These authors contributed equally.

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